

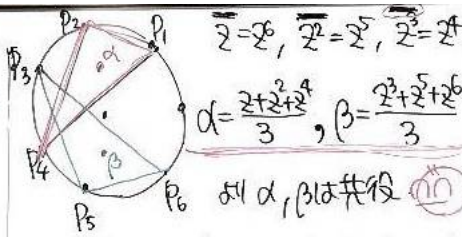
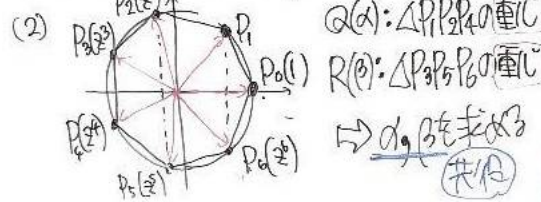
384 $z = \cos \frac{2\pi}{7} + i \sin \frac{2\pi}{7}$ 円方程式

$z^7 = \cos 2\pi + i \sin 2\pi = 1$ (1) (2) (3) の和

$z^7 - 1 = (z-1)(z^6 + z^5 + z^4 + z^3 + z^2 + z + 1) = 0$

$z \neq 1$ かつ $z^6 + z^5 + z^4 + z^3 + z^2 + z + 1 = 0$

(1) $z + z^2 + \dots + z^6 = -1$



$\alpha + \beta = \frac{1}{3}(z+z^2+z^3+z^4+z^5+z^6) = -\frac{1}{3}$

$\alpha\beta = \frac{1}{9} z^2 z^3 (1+z+z^2)(1+z^2+z^3)$
 $= \frac{1}{9} z^5 (1+z+z^2+z^3+z^4+z^5+z^6)$

$= \frac{1}{9} z^5 \cdot z^2 = \frac{z^7}{9} = \frac{1}{9}$

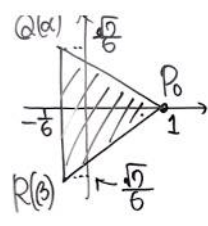
α, β は t の方程式 KKK

$t^2 + \frac{1}{3}t + \frac{1}{9} = 0$ の2解

$9t^2 + 3t + 1 = 0$

$t = \frac{-3 \pm 3\sqrt{3}i}{18} = \frac{-1 \pm \sqrt{3}i}{6}$

例 $\alpha = \frac{-1 + \sqrt{3}i}{6}, \beta = \frac{-1 - \sqrt{3}i}{6}$



$\Delta P_0 Q R$ の面積は
 $\frac{1}{2} \times (1 + \frac{1}{6}) \times (\frac{\sqrt{3}}{6} \times 2)$
 $= \frac{2\sqrt{3}}{36}$

空の
 複素平面
 演習問題が入る

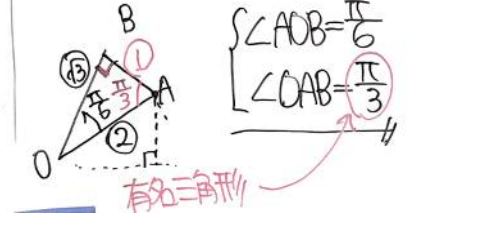
49講義

385 $3\alpha^2 - 6\alpha\beta + 4\beta^2 = 0$ ($t = \frac{\beta}{\alpha}$ とおく)

$4t^2 - 6t + 3 = 0$

(1) $t = \frac{\beta}{\alpha} = \frac{3 \pm \sqrt{3}i}{4} = \frac{\sqrt{3}}{2} \left\{ \cos\left(\pm\frac{\pi}{6}\right) + i \sin\left(\pm\frac{\pi}{6}\right) \right\}$

(2) \vec{OB} は \vec{OA} を $\frac{\sqrt{3}}{2}$ 倍縮小し、 $\pm\frac{\pi}{6}$ 回転したものである



386 $A(z_1 = ti), B(z_2 = u-i), C(z_3 = (b+1) + bi)$

例 ΔABC が正三角形

$\Leftrightarrow \vec{AC}$ (または \vec{AB}) を $\pm\frac{\pi}{3}$ 回転したものである

$\therefore z_3 - z_1 = (z_2 - z_1) \times \left\{ \cos\left(\pm\frac{\pi}{3}\right) + i \sin\left(\pm\frac{\pi}{3}\right) \right\}$

実部虚部を比較

(a, b) = (3, \sqrt{3}), (3, -\sqrt{3})

387 z : 原点中心, 半径2の円上

$\therefore |z| = 2$ z は z の共役

$w = (1-i)z - 2i$

$\Rightarrow w$ の軌跡を求める $\Rightarrow z$ を消去

$z = \frac{w+2i}{1-i}$ かつ $|w - (-2i)| = 2\sqrt{2}$

$\frac{|w+2i|}{|1-i|} = 2$ 中心 $-2i$, 半径 $2\sqrt{2}$ の円

$\frac{|w+2i|}{|1-i|} = 2$ $|\alpha + \beta| = x$
 $|\alpha \cdot \beta| = |\alpha| |\beta|$

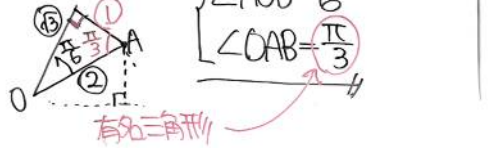
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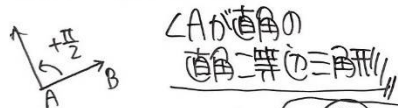
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実部虚部を比較

(a, b) = (3, \sqrt{3}), (3, -\sqrt{3})

$\gamma - d = (\beta - d) \times \cos(\pm \frac{\pi}{2}) + i \sin(\pm \frac{\pi}{2})$

AC は AB を $\pm \frac{\pi}{2}$ 回転したも



∠Aが直角の
直角二等辺三角形



三角形の形状 回転A

$\triangle OAB \Rightarrow \frac{\beta-d}{\alpha-d} \text{ 回転 } O$

$\triangle ABC \Rightarrow \frac{\gamma-d}{\beta-d} \text{ 回転 } A$

388 α, β, γ : 異なる複素数

$2\alpha^2 + \beta^2 + \gamma^2 - 2\alpha\beta - 2\alpha\gamma = 0$

(1) $\frac{\gamma-d}{\beta-d} = \boxed{2} \Rightarrow$ (2) $\triangle ABC$ の形状

(β, γ を 1 箇所 (= まいどおし))
誘導

$\gamma^2 - 2d\gamma + (2d^2 + \beta^2 - 2d\beta) = 0$

$(\gamma-d)^2 + d^2 + \beta^2 - 2d\beta = 0$

$(\gamma-d)^2 + (\beta-d)^2 = 0$

$(\frac{\gamma-d}{\beta-d})^2 + 1 = 0$

$\therefore \frac{\gamma-d}{\beta-d} = \pm i$ (1)

②) は AB を $\pm \frac{\pi}{2}$ 回転したも

∠Aが直角の
直角二等辺三角形

検算
計算
図, 方程式

$x^2 + 3 = 0$

③) k : 実数の定数 $f(z) = kx$

α, β, γ は $x^2 + kx + 20 = 0$ の 3 解

$\Rightarrow \alpha, \beta, \gamma$ おお k を求める

KKK

$\begin{cases} \alpha + \beta + \gamma = 0 \\ \alpha\beta + \beta\gamma + \gamma\alpha = -k \\ \alpha\beta\gamma = -20 \end{cases}$

(1)(2) を誘導しなす

3次関数のグラフが
示す(とも)実数解をも

②) お 1) は実数解, 残りは共役

$\alpha = d$ $\beta = \gamma$

YouTube

$\alpha = p, \beta = (p+q) + qi, \gamma = (p+q) - qi$ かつ

KKK お

$\alpha + \beta + \gamma = 3p + 2q = 0$ (1)

$\alpha\beta + \beta\gamma + \gamma\alpha = \alpha(\beta + \gamma) + \beta\gamma$ (2)

$= p \times 2(p+q) + (p+q)^2 + q^2 = -k$

$\alpha\beta\gamma = p \times \{(p+q)^2 + q^2\} = -20$ (3)

$q = -\frac{3}{2}p$ $\frac{5}{2}p^3 = -20, p^3 = -8$

$q = \frac{3}{4}p$ $p = -2, q = 3$

$(\beta-d)^2 + (\gamma-d)^2 = 0$

以上お

$k = 6, d = -2, \beta = 1 + 3i, \gamma = 1 - 3i$

お k を求める

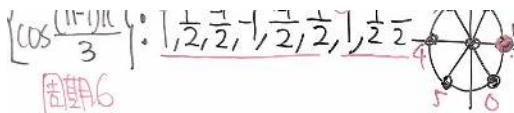
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YouTube



2^{n-1} に着目 ($2^{10} = 1024$) $n \geq 11$ が必要

$\begin{cases} z_{11} = 2^{10} \times (-\frac{1}{2}) = -512 \times \\ z_{12} = 2^{11} \times \frac{1}{2} = 1024 \end{cases}$

求める最小の n は $n = 12$

389) $z_0 = 1 + \frac{\sqrt{3}}{3}i$

(1) $z_{n+1} = (1 + \sqrt{3}i)z_n + 1$

$\rightarrow \alpha = (1 + \sqrt{3}i)\alpha + 1$ ($\alpha = \frac{-1}{\sqrt{3}i}$)

$z_{n+1} - \alpha = (1 + \sqrt{3}i)(z_n - \alpha)$ ($= \frac{2}{\sqrt{3}}$)

$z_{n+1} - \frac{2}{\sqrt{3}} = (1 + \sqrt{3}i)(z_n - \frac{2}{\sqrt{3}})$ ← 等比

漸化式 (等比)

$z_n - \frac{2}{\sqrt{3}} = (z_1 - \frac{2}{\sqrt{3}}) \times (1 + \sqrt{3}i)^{n-1}$


$\therefore z_n = (1 + \sqrt{3}i)^{n-1} + \frac{2}{\sqrt{3}}$

(2) $z_n = 2 \left[\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right]^{n-1} + \frac{2}{\sqrt{3}}$

$= 2^{n-1} \left[\cos \frac{(n-1)\pi}{3} + i \sin \frac{(n-1)\pi}{3} \right] + \frac{2}{\sqrt{3}}$

$\text{Re}(z_n) = 2^{n-1} \times \cos \frac{(n-1)\pi}{3}$ (周期)

$\left\{ \cos \frac{(11-k)\pi}{3} \right\}: 1, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, 1, \frac{1}{2}, -\frac{1}{2}$



解法(1)比等

2^{n-1} (着目) $(2^{10} = 1024)$ $n \geq 11$ が必要

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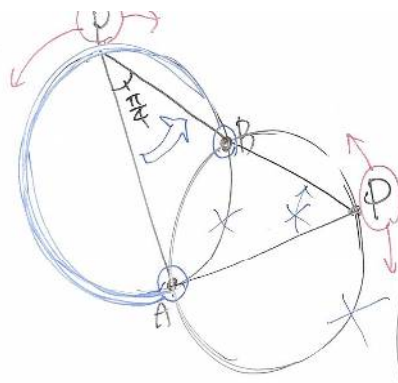
求める最小の n は $n=12$

(1) $z_1 = 1 + \frac{\sqrt{3}}{3}i$
 $z_{n+1} = (1 + \sqrt{3}i)z_n + 1$

$\rightarrow \alpha = (1 + \sqrt{3}i)\alpha + 1$ $\left(\alpha = \frac{-1}{\sqrt{3}i} \right)$
 $z_{n+1} - \alpha = (1 + \sqrt{3}i)(z_n - \alpha)$ $\left(= \frac{2}{\sqrt{3}} \right)$
 $z_{n+1} - \frac{2}{\sqrt{3}} = (1 + \sqrt{3}i)(z_n - \frac{2}{\sqrt{3}})$ ← 等比

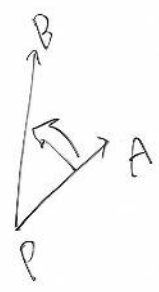
$z_n - \frac{2}{\sqrt{3}} = (z_1 - \frac{2}{\sqrt{3}}) \times (1 + \sqrt{3}i)^{n-1}$
 $\therefore z_n = (1 + \sqrt{3}i)^{n-1} + \frac{\sqrt{3}}{3}i$

(2) $z_n = \left\{ 2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) \right\}^{n-1} + \frac{\sqrt{3}}{3}i$
 $= 2^{n-1} \times \left\{ \cos \frac{(n-1)\pi}{3} + i \sin \frac{(n-1)\pi}{3} \right\} + \frac{\sqrt{3}}{3}$
 $\text{Re}(z_n) = 2^{n-1} \times \left\{ \cos \frac{(n-1)\pi}{3} \right\}$ 周期



\vec{PB} は \vec{PA} を $\frac{1}{2}$ 回転
 拡大縮小したもの

$\phi(C)$
 $r = 3 + 4i$
 $\angle = 28^\circ$



(1) $A(\alpha = 6), B(\beta = 11), P(z = \frac{1}{(-1-i)(-1-i)})$ の

(1) $\angle APB$ の大きさを
 $\frac{\vec{PB}}{\vec{PA}} = \frac{\beta - z}{\alpha - z} = \frac{11 - z}{6 - z} = \frac{11 - \frac{1}{(-1-i)(-1-i)}}{6 - \frac{1}{(-1-i)(-1-i)}} = \dots$
 $= \frac{\sqrt{12}}{2L} (\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$

